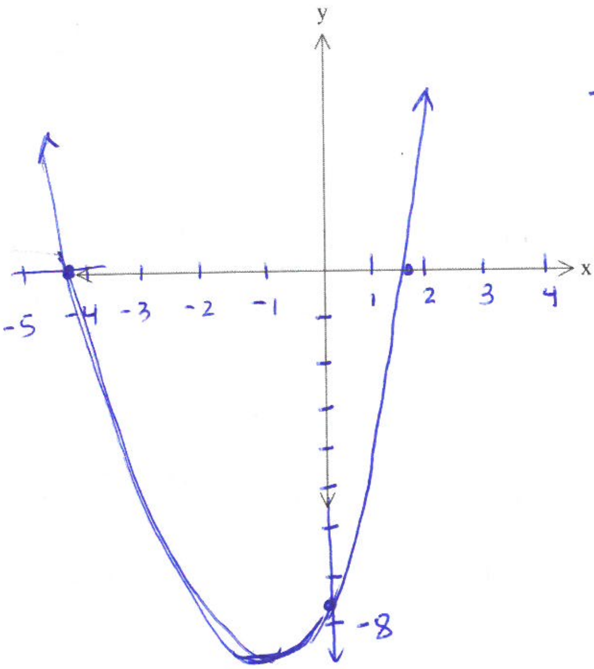
Example 2: Sketching Quadratic Functions using Intercepts

Sketch the graph of $y = x^2 + 2.25x - 7.88$. Label the y intercept and the x intercepts.



x int
1.90
-4.15

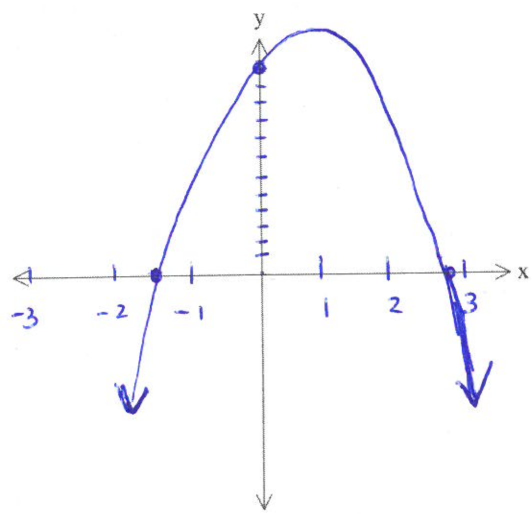
y int
= -7.88

x-int,
1) $Y_2 = 0$
2) $\boxed{2ND}$ $\boxed{TRACE}$
$\boxed{5}$ Intersect
3) $\boxed{ENTER}$ 3 times.

y-int
1) $\boxed{2ND}$ $\boxed{TRACE}$
2) $\boxed{1}$ value
3) type in $X = 0$
4) $\boxed{ENTER}$

Example 3

Sketch the graph of $y = -2.65x^2 + 3.9x + 12$. Label the y intercept and the x intercepts.

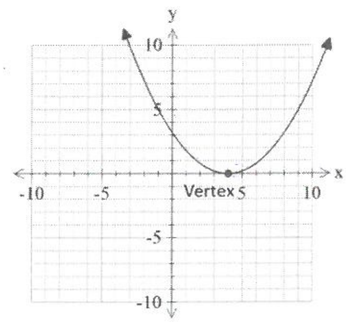


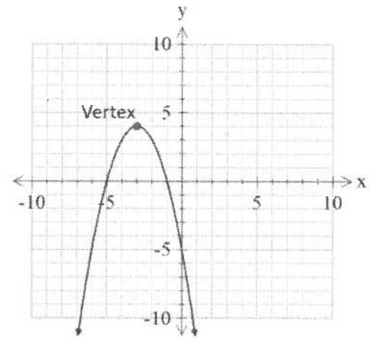
X-int -1.52
2.99

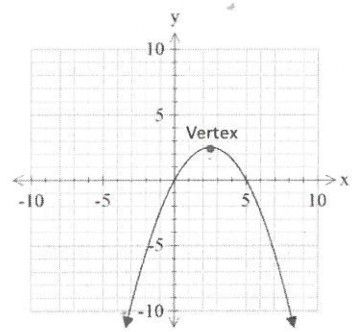
y int 12

Characteristics of Quadratic Functions

The ***vertex*** of a parabola is the point where the direction of the graph turns (sometimes called the turning point). If you draw a vertical line through the vertex you will see that the parabola is perfectly symmetrical.



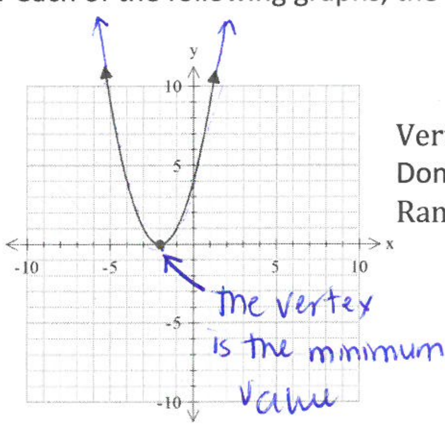




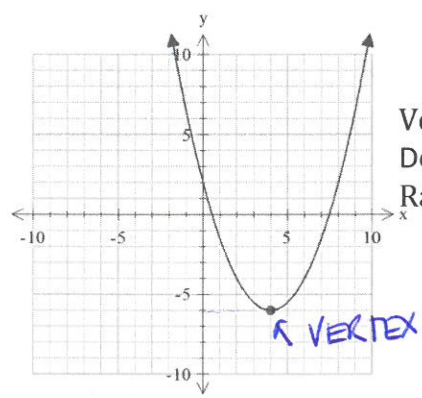
The vertex is an important point of the graph because it is where the **maximum** or **minimum** value occurs. When graphing or analyzing quadratic functions, we should always examine the vertex.

If we know the vertex and the direction of a parabola, then we know a lot about the location of the function.
We can see that the domain of a quadratic function is limitless, like the linear function. But the range of a quadratic function is limited by the location of the vertex and the direction of the opening.

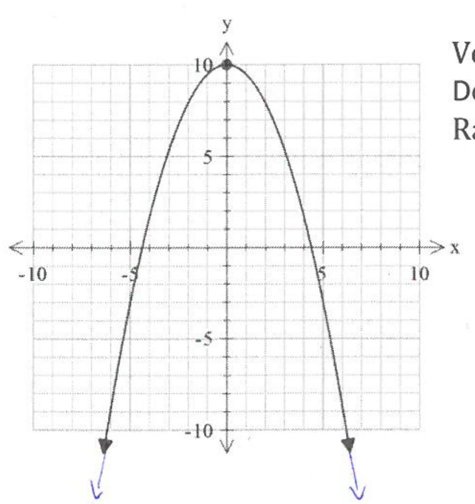
For each of the following graphs, the vertex and range are shown.



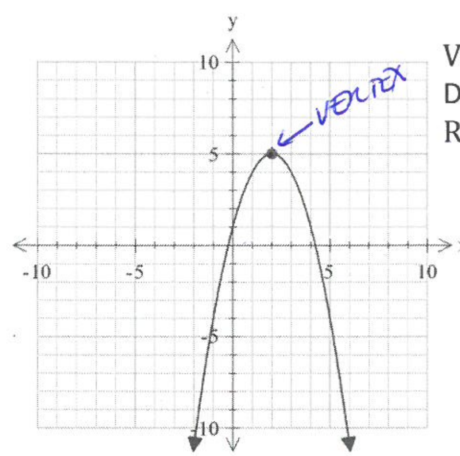
Vertex: $(-2, 0)$
Domain: $x \in R$
Range: $[0, \infty)$



Vertex: $(4, -6)$
Domain: $x \in R$
Range: $[-6, \infty)$



Vertex: $(0, 10)$
Domain: $x \in R$
Range: $(-\infty, 10]$



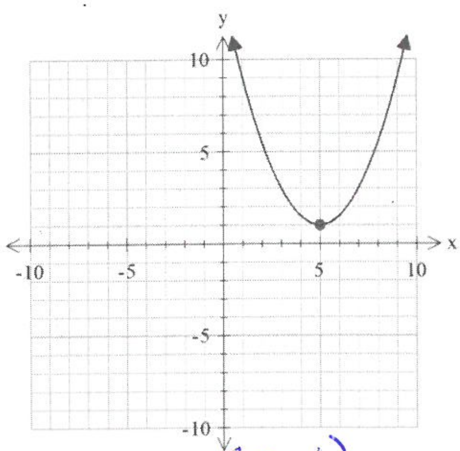
Vertex: $(2, 5)$
Domain: $x \in R$
Range: $(-\infty, 5$

$\in$ means element of

$\mathbb{R}$ means real number.

Example 1: Analyzing Quadratic Functions

State the vertex and the range for the following functions.



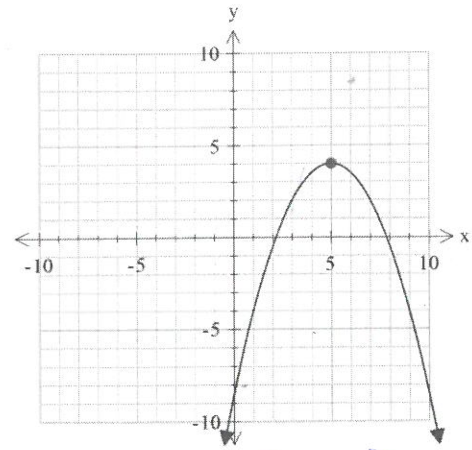
Vertex: $(5, 1)$

ALL x-values → Domain: $x \in \mathbb{R}$

Range: $[1, \infty)$

could reach 1

could not reach the ∞



Vertex: $(5, 4)$

Domain: $x \in \mathbb{R}$

Range: $(-\infty, 4]$

In addition to stating the vertex and the domain and range of a quadratic function, many other characteristics can be described such as end behaviour and x and y intercepts.

The quadratic function shown below can be described as having the following characteristics:

Shape: Parabola opening upward.

Sign of leading coefficient: Positive

End Behaviour: Q II to Q I.

y-intercept: 0.

x-intercepts: -4 and 0.

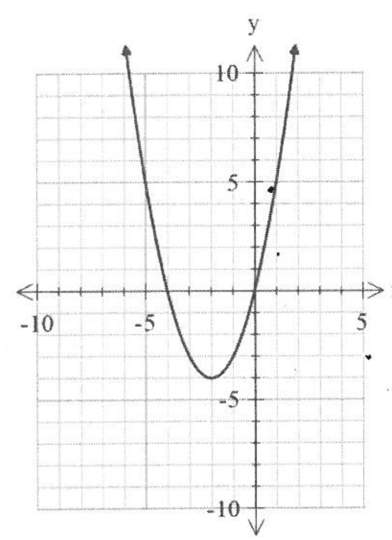
Vertex: $(-2, -4)$

Minimum Value: -4

Maximum Value: ~~None~~ ∞

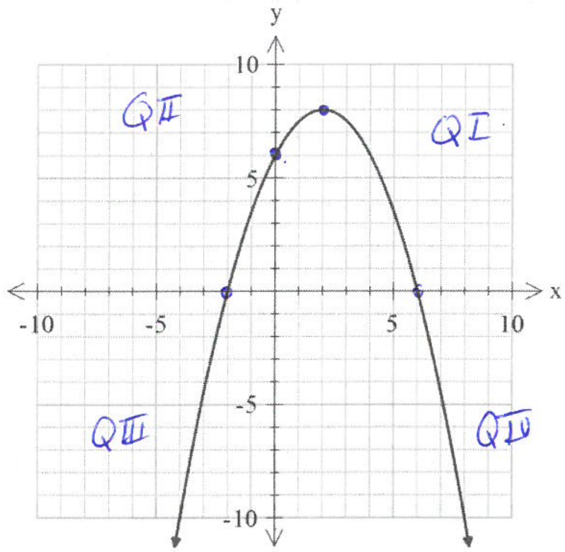
Domain: $x \in R$

Range: $[-4, \infty)$



Using the vocabulary from Example 1, fill in the blanks for the quadratic functions in Examples 2 and 3.

Example 2



Direction of opening: down

Sign of leading co-efficient: Neg.

End behaviour: QIII to QIV

y-intercept: 6

x-intercept(s): -2 and 6

Vertex: (2, 8)

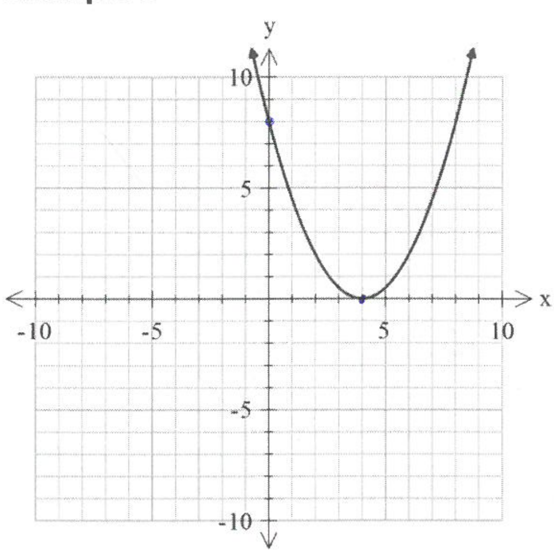
Minimum value: $-\infty$

Maximum value: 8

Domain: $x \in \mathbb{R}$

Range: $(-\infty, 8]$

Example 3



Direction of opening: UP

Sign of leading co-efficient: Positive

End behaviour: QII to QI

y-intercept: 8

x-intercept(s): 4

Vertex: (4, 0)

Minimum value: 0

Maximum value: ∞

Domain: $x \in \mathbb{R}$.

Range: $[0, \infty)$

Example 5: Using the Graphing Calculator to determine Characteristics of Quadratic Functions

Analyze each quadratic function by providing the requested characteristics.

a) $y = 0.5x^2 - 2x + 7$

Direction of opening: UP

End behaviour: QII to QI

x-intercept(s): None

Minimum value: 5

Domain: $x \in \mathbb{R}$

Sign of leading co-efficient: Positive

y-intercept: 7

Vertex: (2, 5)

Maximum value: ∞

Range: $[5, \infty)$

to determine the VERTEX

$\boxed{\text{2ND}}$ $\boxed{\text{TRACE}}$

$\boxed{3}$ for minimum

OR

$\boxed{4}$ for maximum

use ◀ and ▶ to set boundaries

$\boxed{\text{ENTER}}$

b) $y = (7 - 2x)(x + 10)$

Direction of opening: DOWN

End behaviour: Q III to Q IV

x-intercept(s): 3.5 and 10

Minimum value: $-\infty$

Domain: $x \in \mathbb{R}$

Sign of leading co-efficient: NEGATIVE

y-intercept: 70

Vertex: (−3.25, 91.125)

Maximum value: 91.125

Range: $(-\infty, 91.125]$

c) $y = -34.2x^2 + 8.25x - 44$

Direction of opening: DOWN

End behaviour: Q III to Q IV

x-intercept(s): NONE

Minimum value: $-\infty$

Domain: $x \in \mathbb{R}$

Sign of leading co-efficient: NEGATIVE

y-intercept: −44

Vertex: (0.121, −43.502)

Maximum value: −43.502

Range: $(-\infty, -43.502]$